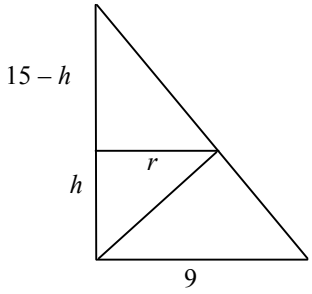
Assessment Schedule – 2005

Calculus: Differentiate and use derivatives to solve problems (90635)

Evidence Statement

	Achievement Criteria	Q	Evidence	Code	Judgement	Sufficiency
Achievement	Differentiate functions and use differentiation to solve problems.	1(a)	$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} + 35e^{7x}$	A1	Or equivalent.	**Achievement:** 3 × code A including **at least** 1 × code A1 and 1 × code A2.
		1(b)	$\frac{dy}{dx} = \frac{1}{\sin x}.\cos x$	A1	Or equivalent.	
		2	$y = \cos x$ $\frac{dy}{dx} = -\sin x$ When $x = \frac{\pi}{6}$, $y = \frac{\sqrt{3}}{2}$ and $\frac{dy}{dx} = -\frac{1}{2}$ $y - \frac{\sqrt{3}}{2} = -\frac{1}{2}(x - \frac{\pi}{6})$ or $x + 2y - \sqrt{3} - \frac{\pi}{6} = 0$ or $x + 2y - 2.26 = 0$ or $y = \frac{-1}{2}x + \frac{\sqrt{3}}{2} + \frac{\pi}{12}$ or $y = \frac{-1}{2}x + 1.128$	A2	Or equivalent.	
		3	$\frac{dR}{dt} = \frac{dR}{dx}.\frac{dx}{dt}$ $= (42 - \frac{x}{9}) \times 20$ when $x = 300$, $\frac{dR}{dt} = \$173\frac{1}{3}$ per day	A2	Or equivalent. Accept \$173. Units not required.	

	Achievement Criteria	Q	Evidence	Code	Judgement	Sufficiency
Achievement with Merit	Demonstrate knowledge of concepts and techniques of differentiation.	4	$\frac{dy}{dx} = \frac{e^{3x}.4x - (2x^2+1)3e^{3x}}{(e^{3x})^2}$	A1, M1	Or equivalent.	**Achievement with Merit:** **EITHER** As for Achievement **plus** 2 × code M1 **and** 2 × code M2 **OR** 6 × code M
		5	$\frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx}$ $= \frac{7\sec^2 t}{5\sec t \tan t}$ or in Cartesian form: $\frac{x^2}{25} - \frac{y^2}{49} = 1 \rightarrow \frac{dy}{dx} = \frac{49x}{25y}$	A1, M1	Or equivalent $\frac{7\sec t}{5\tan t}, \frac{7}{5\sin t}$ $\frac{7}{5}\text{cosec } t$.	
		6	Point of inflection when: $x = \pm\frac{2}{\sqrt{3}}$ or ± 1.15 Concave down: $-1.15 < x < 1.15$	A1, M1	Accept $-1.15 \le x \le 1.15$ as MEI. Or equivalent.	
	Solve differentiation problems.	7	$C(x) = 17x$ $P(x) = 25x - 20x\ln x$ $P'(x) = 5 - 20\ln x$ Maximum profit when $P'(x) = 0$ $x = e^{0.25}$ $x = 1.284$ For max profit need 1284 T-shirts.	A2, M2	Must see correct derivative in any form. Units not required. Or equivalent.	
		8	$x = 30\tan\theta$ $\frac{dx}{d\theta} = 30\sec^2\theta$ $\frac{dx}{dt} = -3$ $\frac{d\theta}{dt} = \frac{1}{30}\cos^2\theta \times (-3)$ when $\theta = \frac{\pi}{4}$, $\frac{d\theta}{dt} = -\frac{1}{20}$ or -0.05 rad per sec	A2, M2	Accept use of $\frac{dx}{dt} = 3$ to give answer of $\frac{1}{20}$ as MEI. Units not required. Or equivalent.	
		9	(a) $t = 2.5$ seconds (b) $a = -10\ \text{ms}^{-2}$	A2, M2	Units not required. Or equivalent. Need both (a) and (b).	

	Achievement Criteria	Q	Evidence	Code	Judgement	Sufficiency
Achievement with Excellence	Solve problem(s) involving a combination of differentiation techniques.	10	 Similar triangles gives: $\frac{15-h}{r}=\frac{15}{9}$ $h=15-\frac{5}{3}r$ $V=\frac{1}{3}\pi r^2 h$ $=\frac{1}{3}\pi r^2(15-\frac{5}{3}r)$ $\frac{dV}{dr}=10\pi r-\frac{5}{3}\pi r^2$ Turning points when: $\frac{dV}{dr}=0$ $r(6-r)=0$ when $r=0, 6$. When $r=0$, $V=0$, is a minimum, so maximum when $r=6$. Therefore the dimensions of the smaller cone are $r=6$ and $h=5$. The maximum volume of the smaller cone will be 188.5 cm^3 (60π). **Optional second derivative test:** $\frac{d^2V}{dr^2}=10\pi-\frac{30}{9}\pi r$ When $r=6$, $\frac{d^2V}{dr^2}=-31.4$ $\frac{d^2V}{dr^2}<0$, so $r=6$ gives max. volume.	A, M, E	Need correct first derivative. Need dimensions of smaller cone that gives max. volume. Units not required.	**Achievement with Excellence:** As for Merit **plus** code E.

Judgement Statement

Achievement	Achievement with Merit	Achievement with Excellence
Differentiate functions and use differentiation to solve problems. 3 × A *including* at least 1 × A1 *and* at least 1 × A2	Demonstrate knowledge of concepts and techniques of differentiation. Solve differentiation problems. Achievement *plus* 2 × M1 *and* 2 × M2 ***or*** 6 × M	Solve problem(s) involving a combination of differentiation techniques. Merit *plus* 1 × E